Leibniz: Jurnal Matematika
Januari 2026
Vol. 6 No. 01 Hal. 29-38
e-ISSN: 2775-2356
https://ejurnal.unisap.ac.id/leibniz/index

DOMINATING SETS AND CONNECTIVITY PRESERVATION IN POWER GRAPHS OF SYMMETRIC AND CYCLIC GROUPS

Ika Metiza Maris[1,2], Rawdah Adawiyah Tarmizi[1*], Nor Hafizah Md Husin[1]

[1] Department of Mathematics, Universiti Pendidikan Sultan Idris, Perak Darul Ridzuan, Malaysia
[2] Department of Tadris Mathematics, UIN Mahmud Yunus Batusangkar, Sumatera Barat, Indonesia
**email: rawdah@fsmt.upsi.edu.my*

Abstract: The power graph $\mathcal{P}(G)$ is a simple graph associated with a group G that represents power relations among its elements. Although power graphs have been widely studied in connection with domination and connectivity, the effect of removing dominating sets, particularly those excluding the identity element, on graph connectivity has not been examined in detail. This study aims to characterize dominating sets in power graphs of finite groups and to investigate whether connectivity is preserved after their removal, with emphasis on symmetric groups and cyclic groups. This research employs a theoretical and analytical approach based on group theory and algebraic graph theory. The results show that, for symmetric groups S_n, there exists a dominating set excluding the identity element such that the power graph remains connected after its removal. Furthermore, for cyclic groups $\mathbb{Z}_n$, any generator forms a minimum dominating set, and t $\mathcal{P}(\mathbb{Z}_n)$ remains connected after its removal.

Keywords: power graph; dominating set; symmetric group; cyclic group; connectivity

Abstrak: *Graf pangkat $\mathcal{P}(G)$ adalah graf sederhana yang berkaitan dengan suatu grup hingga G dan merepresentasikan relasi perpangkatan antar elemennya. Meskipun graf pangkat telah banyak diteliti dalam kaitannya dengan dominasi dan konektivitas, pengaruh penghapusan himpunan dominasi, khususnya yang tidak memuat elemen identitas, terhadap konektivitas graf belum dikaji secara mendalam. Penelitian ini bertujuan untuk mengkarakterisasi himpunan dominasi dalam graf pangkat dari grup hingga serta meneliti apakah konektivitas graf tetap terpelihara setelah penghapusan himpunan dominasi tersebut, dengan penekanan pada grup simetri dan grup siklik. Penelitian ini menggunakan pendekatan teoretis dan analitis yang berlandaskan teori grup dan teori graf aljabar. Hasil penelitian menunjukkan bahwa, untuk grup simetri S_n, terdapat himpunan dominasi yang tidak memuat elemen identitas sehingga graf pangkat tetap terhubung setelah penghapusannya. Selain itu, untuk grup siklik $\mathbb{Z}_n$, setiap generator merupakan himpunan dominasi minimum dan setelah penghapusan himpunan dominasi minimum $\mathcal{P}(\mathbb{Z}_n)$ tetap terhubung.*

Kata Kunci: *graf pangkat; himpunan dominasi; grup simetri; grup siklik; konektivitas*

Leibniz: Jurnal Matematika
Januari 2026
Vol. 6 No. 01 Hal. 29-38
e-ISSN: 2775-2356
https://ejurnal.unisap.ac.id/leibniz/index

INTRODUCTION

The study of power graphs forms a natural bridge between algebra and graph theory by encoding algebraic properties of groups into graph-theoretic structures. The power graph $\mathcal{P}(G)$ of a group G is a graph whose vertices correspond to the elements of G, with edges connecting two vertices x and y if one is a power of the other. This representation encapsulates subgroup relationships and structural dominance within finite groups (Kelarev & Quinn, 2002; Manju, 2025). The power graph has been widely studied concerning its connectivity, domination properties, clique number, and automorphism group (Aalipour et al., 2017; Abawajy et al., 2013; Asmarani et al., 2022; Cameron & Ghosh, 2011; Hamzeh, 2021).

A fundamental aspect of power graph research is the dominating set, which consists of a subset of vertices such that every other vertex in the graph is adjacent to at least one element of the set. Understanding minimal and maximal dominating sets is critical for analyzing the structural robustness of $\mathcal{P}(G)$. Earlier studies have examined dominating sets in cyclic groups, dihedral groups, and symmetric groups, focusing on their size, uniqueness, and diameter (Doostabadi & Erfanian, 2013; Malarvizhi & Revathi, 2017; Pourghobadi & Jafari, 2018). However, the characterization of dominating sets in power graphs of symmetric groups $\mathcal{P}(S_n)$, and cyclic groups $\mathcal{P}(\mathbb{Z}_n)$ remains an active area of research.

The cyclic group $\mathbb{Z}_n$, as one of the simplest algebraic structures, provides a foundational case for studying power graphs. In $\mathcal{P}(\mathbb{Z}_n)$ the adjacency of vertices reflects the divisors of n, and specific subsets of elements dominate the graph. Research has shown that dominating sets in $\mathbb{Z}_n$ exhibit unique patterns based on the groups order and structure(Cameron & Ghosh, 2011; Chattopadhyay, 2019). Prior research has established that the connectivity of $\mathcal{P}(\mathbb{Z}_n)$ depends on the existence of dominant vertices, elements whose power relations ensure adjacency to a significant portion of the graph (Chakrabarty et al., 2009; Doostabadi & Ghouchan, 2015; Mehranian et al., 2016).

The symmetric group S_n, representing all permutations of n elements, offers a rich combinatorial structure for exploring power graphs. In $\mathcal{P}(S_n)$, dominating sets arise naturally due to the relationships between elements of various subgroups. Cameron (2011) established that $\mathcal{P}(S_n)$, is always connected, implying that each element is power-adjacent to at least one other element. However, the characterization of maximal dominating sets, particularly those excluding the identity element, has remained an open problem. Studies on proper power graphs, where the identity is removed, suggest that connectivity may still be preserved under certain conditions (Doostabadi & Ghouchan, 2015). However, the structure and impact of maximal dominating sets, particularly those that exclude the identity element, have received limited attention. In particular, it remains unclear whether the removal of such dominating sets necessarily disrupts the connectivity of $\mathcal{P}(S_n)$.

Recent studies have also focused on unique minimal dominating sets, which ensure that the power graph remains structurally sound while minimizing redundancy (Malarvizhi & Revathi, 2017). In certain graph classes, including power graphs, these sets play a crucial role in maintaining connectivity and optimizing network properties. While several results exist for semigroups and abelian groups (Abawajy et al., 2013),(Chakrabarty et al., 2009), their implications for non-abelian structures such as S_n and Q_{2n} remain underexplored. Despite these findings, the characterization of maximal dominating sets in $\mathcal{P}(S_n)$, particularly those excluding the identity element, have received limited attention. Addressing this gap requires a detailed

Leibniz: Jurnal Matematika
Januari 2026
Vol. 6 No. 01 Hal. 29-38
e-ISSN: 2775-2356
https://ejurnal.unisap.ac.id/leibniz/index

analysis of the adjacency relationships within $\mathcal{P}(S_n)$, and the implications of removing dominating sets on the graph's connectivity.

This paper focuses on the role of dominating sets that exclude the identity element in the power graphs of symmetric and cyclic groups. Specifically, we construct maximal dominating sets in $\mathcal{P}(S_n)$and show that, despite their removal, the resulting power graphs remain connected. In addition, we analyze dominating sets in $\mathcal{P}(\mathbb{Z}_n)$ to clarify the relationship between generators, domination, and connectivity. These results reveal a concrete mechanism of structural resilience in power graphs and provide new insight into how algebraic subgroup structure influences domination and connectivity in algebraic graph theory.

METHODOLOGY

This study employs a theoretical and analytical approach based on concepts from group theory and algebraic graph theory. The main objects of study are power graphs of finite groups, particularly symmetric groups and cyclic groups, together with their dominating sets. Power graphs are constructed by taking group elements as vertices and defining adjacency through power relations. Dominating sets are identified using a constructive method, based on the structure of cyclic subgroups and element orders. For symmetric groups, maximal dominating sets excluding the identity element are explicitly constructed, while for cyclic groups, dominating sets are characterized using group generators. The effect of dominating set removal is examined through connectivity analysis of the resulting graph. Connectivity is established by proving the existence of paths between remaining vertices using subgroup containment and power relations. All results are obtained through rigorous mathematical proofs, supported by illustrative examples for small groups.

To support this analysis, the study first introduces the fundamental definitions and theoretical results from group theory and graph theory that underpin the construction of power graphs and the notion of domination. These preliminary concepts provide the essential framework for understanding dominating sets and the structural properties of power graphs of finite groups, and they form the basis for the main results developed in the subsequent section.

Definition 1. (Davvaz, 2021) The symmetric group S_n consist of all bijections function on a finite set containing n distinct elements. Formally, it is defined as

$$S_n = \{\alpha : \{1, 2, 3, \dots, n\} \longrightarrow \{1, 2, 3, \dots, n\} | \alpha \text{ is bijection}\}.$$

Here α, β represents a permutation, and the group operation is composition if $\alpha, \beta \in S_n$, then $\alpha \circ \beta \in S_n$.

Definition 2. (Davvaz, 2021) A group G is called cyclic if there exists an element $a \in G$ such that $G = \langle a \rangle$. Such an element a is called a generator of G.

Definition 3. (Parveen et al., 2024) Let graph G be defined as an ordered pair $\Gamma = (V, E)$ where $V(\Gamma)$ represent the set of vertices and $E(\Gamma)$ is a set of edges of G. A vertex u is called a dominating vertex of the graph G if for all $v \in V(\Gamma)$ with $v \neq u$, there exists an edge $(u, v) \in E(\Gamma)$.

Definition 4. (Haynes et al., 1998; Panda & Krishna, 2018; S. Shukla, 2020) For $S \subseteq V$ of vertices in a graph $\Gamma = (V, E)$ is called a dominating set if every vertex $u \in V$ is either an element of S or is adjacent to an element of S. Furthermore, a dominating set S is a minimal dominating set if no proper subset $S'' \subset S$ is a dominating set. Additionally, among all minimal dominating sets, the one with the smallest cardinality is referred to as the minimum dominating set.

Definition 5. (Marcus, 2020) A graph is connected if every vertex is joined to every other vertex by a path.

RESULT AND DISCUSSION

Result

In this section, we present the findings of our research on the structure of dominating sets in symmetric and cyclic groups. Additionally, we provide a rigorous analysis demonstrating that the removal of a specific dominating set does not compromise the connectivity of the group's power graph.

3.1 Symmetric Group

In the power graph of S_n, a trivial dominating set exists, consisting solely of the identity element. As the identity element is adjacent to every other element in S_n,, it serves as a universal vertex, ensuring the graph's overall connectivity. Consequently, removing this element would disconnect the power graph.

Lemma 1. Let S_n be symmetric group on n element, and let $\mathcal{P}(S_n)$ be the power graph of S_n. Let e denote the identity vertex of $\mathcal{P}(S_n)$. Then $\mathcal{P}(S_n) - \{e\}$ is disconnected.

Proof. Let $\mathcal{P}(S_n)$ be defined by a graph where each vertex corresponds to an element of S_n, and two vertices g and h are adjacent if $h^k = g$ for some positive integer k. In particular, the identity element e of S_n is adjacent to every vertex in $\mathcal{P}(S_n)$, making it a dominating vertex.

Now, consider $g, h \in \mathcal{P}(S_n)$. If $h^k = g$ for some positive integer k then lies in the cyclic subgroup generated by h, denoted $\langle h \rangle$. Similarly, if $g^m = h$ for some positive integer m. In both cases, g and h belong to the same cyclic subgroup, and hence g and h are directly adjacent in $\mathcal{P}(S_n)$.

Suppose g and h are belong to different cyclic subgroups. The only common neighbor between them is the identity element e, as e is adjacent to all vertices in $\mathcal{P}(S_n)$. Hence, there is a path from g to h in $\mathcal{P}(S_n)$ passing through e. If e is removed from $\mathcal{P}(S_n)$, a path from g to h that relied on e as a dominating vertex no longer exists. Consequently, g and h are disconnected in $\mathcal{P}(S_n) - \{e\}$. Therefore, the graph $\mathcal{P}(S_n) - \{e\}$ is disconnected. ■

In the following discussion, we analyze the impact of other dominating sets on the connectivity of the power graph of S_n. In more refined analyses of connectivity, it becomes necessary to examine dominating sets whose influence extends beyond the trivial dominance of the identity element. To formalize this broader perspective, we introduce the notion of a maximal dominating set.

Definition 6. A maximal dominating set, denoted as D_{max}, is defined as a dominating set with the largest cardinality among all minimal dominating sets, excluding the identity element.

Theorem 2. Let S_n be symmetric group on n element, and let $\mathcal{P}(S_n)$ denote the power graph of S_n. There exists a maximal dominating set $D_{max} \subseteq S_n$, excluding the identity element e, such that if D_{max} is removed from $\mathcal{P}(S_n)$, the graph $\mathcal{P}(S_n) - D_{max}$ remain connected.

Proof. Let S_n be the symmetric group on n elements. The group S_n contains many cyclic subgroups of varying orders. For any cyclic subgroup $\langle g \rangle \subseteq S_n$, generated by an element $g \in S_n$, where $\langle g \rangle = \{g^k \in S_n | k \in \mathbb{Z}\}$. This subgroup consists of all powers of g under the group

Leibniz: Jurnal Matematika
Januari 2026
Vol. 6 No. 01 Hal. 29-38
e-ISSN: 2775-2356
https://ejurnal.unisap.ac.id/leibniz/index

operation. In the power graph $\mathcal{P}(S_n)$, vertices correspond to the element of S_n, and two vertices are adjacent if one is a power of the other. For every element $h \in \langle g \rangle$, there exists an integer k such that $g^k = h$. This implies that g is adjacent to every $h \in \langle g \rangle$ in $\mathcal{P}(S_n)$. Consequently, g dominates all vertices of its cyclic subgroup $\langle g \rangle$ in $\mathcal{P}(S_n)$.
To construct a maximal dominating set $D_{max} \subseteq S_n$, one generator is selected from each cyclic subgroups $\langle g \rangle$ in S_n, excluding the identity element. This ensures that D_{max} dominates all vertices in $\mathcal{P}(S_n)$, as each cyclic subgroup $\langle g \rangle$ is fully dominated bay its generator. The identity element e not included in D_{max}, since it is adjacent to every vertex in $\mathcal{P}(S_n)$ and hence provide connectivity regardless of D_{max}.
Next, consider the graph $\mathcal{P}(S_n) - D_{max}$. When D_{max} is removed, the identity element e remains in $\mathcal{P}(S_n)$. Since e is adjacent to every vertex in $\mathcal{P}(S_n)$It acts as a dominating vertex, ensuring connectivity among the remaining vertices. Within each cyclic subgroup $\langle g \rangle$, the elements $h \in \langle g \rangle \setminus \{g\}$ remain connected to e, preserving the structure of $\mathcal{P}(S_n)$.
Therefore, $\mathcal{P}(S_n) - D_{max}$ remains connected. Moreover, the set D_{max} constructed by selecting one generator from each cyclic subgroup, is a maximal dominating set of $\mathcal{P}(S_n)$ that satisfies the condition of the theorem. ■

For $n = 4$, consider the symmetric group $S_4 = \{a_0, a_1, a_2, \ldots, a_6, b_1, \ldots, b_8, c_1, \ldots, c_9\}$. The corresponding power graph of S_4 is illustrated in Figure 1. Define the maximal dominating set of S_4 as $D_{max} = \{a_1, a_2, \ldots, a_6, b_1, b_2, b_3, b_6, c_1, c_4, c_7\}$, where each element in D_{max} serves as the generator of S_4. Upon removing D_{max} from the power graph of S_4, the resulting power graph remains connected, as shown in Figure 2.

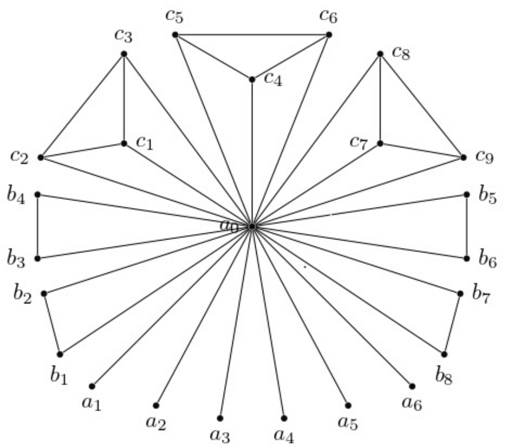

Figure 1. The Power Graph of S_4

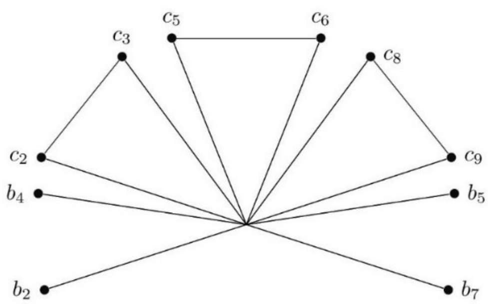

Figure 2. The Power Graph of S_4 without D_{max}

Figure 2 illustrates the power graph of S_4 after removing the maximal dominating set D_{max}. Despite the removal, the identity vertex remains and preserves connectivity among the remaining cyclic subgroups.

3.2 Cyclic Group

In a cyclic group, the dominating set is formed from the group's generators, as they establish adjacency with all other elements in the power graph. The following theorem shows that a single generator is sufficient to form a minimal dominating set. Minimal dominating set of $\mathcal{P}(G)$ denotes by $MDS(\mathcal{P}(G))$ (Haynes et al., 2023).

Theorem 3. Let $\mathbb{Z}_n$ be a cyclic group of order n. The dominating set of a power graph $\mathcal{P}(\mathbb{Z}_n)$ is the set of generators of $\mathbb{Z}_n$. Furthermore, any single generator $g \in \mathbb{Z}_n$ forms a minimum dominating set of $\mathcal{P}(\mathbb{Z}_n)$.

Proof. Consider the cyclic group $\mathbb{Z}_n$ of order n, with elements $\{0, 1, 2, \dots, n-1\}$ and the addition modulo n as the group operation. The power graph $\mathcal{P}(\mathbb{Z}_n)$ is defined such that its vertices correspond to the elements of $\mathbb{Z}_n$, and two vertices g and h are adjacent if $g^k = h$ and $h^m = g$ for some integer k and m.

The identity element 0 in $\mathbb{Z}_n$ is only connected to itself in $\mathcal{P}(\mathbb{Z}_n)$, as $0^k = 0$ for all integers k. On the other hand, each nonzero element g generates the entire elements of group $\mathbb{Z}_n$, forming the cyclic subgroup $\langle g \rangle = \{0, g, 2g, \dots, (n-1)g\}$. In the power graph $\mathcal{P}(\mathbb{Z}_n)$, the vertex corresponding to g is adjacent to all element of $\langle g \rangle$.

The set of generators of $\mathbb{Z}_n$ consists of nonzero elements g such that $\gcd(g, n) = 1$. Therefore, every generator element dominates all the vertices of $\langle g \rangle$ in $\mathcal{P}(\mathbb{Z}_n)$. The set $\{g\}$, where g is any generator of $\mathbb{Z}_n$, forms a minimum dominating set of $\mathcal{P}(\mathbb{Z}_n)$. Including more than one generator would make the dominating set redundant, as any single generator already dominates all nonzero elements, along with the identity element 0, which is inherently connected to all vertices. Hence, the minimum dominating set of $\mathcal{P}(\mathbb{Z}_n)$ consist of any single generator g of $\mathcal{P}(\mathbb{Z}_n)$, demonstrating that the minimum dominating set is not the set of all generators but rather any single one of them. ■

Theorem 4. Let $\mathbb{Z}_n$ be a cyclic group of order n, and let x be a vertex of the power graph $\mathcal{P}(\mathbb{Z}_n)$ such that the singleton set $\{x\}$ is a minimal dominating set of $\mathcal{P}(\mathbb{Z}_n)$.Then the deletion of x is the resulting subgraph $\mathcal{P}(\mathbb{Z}_n)\backslash\{x\}$ remains connected.

Proof. If $n = 1$, then $\mathcal{P}(\mathbb{Z}_n)$ consists of a single vertex, and the result holds trivially.

Suppose first that $n = p^m$ for some prime p and integer $m \geq 1$. In this case, the power graph $\mathcal{P}(\mathbb{Z}_n)$ is complete. Removing any vertex from a complete graph yields another complete graph, which is connected. Hence, $\mathcal{P}(\mathbb{Z}_n)\backslash\{x\}$ remains connected.

Now assume that n is not a prime power and that $\{x\}$ is a minimal dominating set of $\mathcal{P}(\mathbb{Z}_n)$. In this situation, x must be a generator of $\mathbb{Z}_n$; otherwise, there would exist elements not adjacent to x, contradicting the domination property.

In the power graph $\mathcal{P}(\mathbb{Z}_n)$, the identity element e is adjacent to every vertex, since for any $y \in \mathbb{Z}_n$, we have $e = y^{|y|}$. Thus, e is a universal vertex.

After removing the generator x, the identity element e remains in the graph and is still adjacent to all remaining vertices. Therefore, for any two distinct vertices $u, v \in \mathcal{P}(\mathbb{Z}_n)$, there exists a path $u \sim e \sim v$ showing that all vertices lie in the same connected component. ■

In the case of $n = 6$, the cyclic group $\mathbb{Z}_6 = \{0, 1, 2, 3, 4, 5\}$ serves as an example of a non-prime-order group. The corresponding power graph of $\mathbb{Z}_6$ is depicted in Figure 3. Since 6 is not a prime number, based on Theorem 3 the generator of $\mathbb{Z}_6$, namely $\{1\}$ and $\{5\}$, form the minimal dominating set of $\mathcal{P}(\mathbb{Z}_6)$. If either $\{1\}$ or $\{3\}$ is removed, the power graph remains connected because the identity element continues to dominate all other vertices. Another minimal dominating set can be found in $\mathcal{P}(\mathbb{Z}_6)$ that does not consist of its generators. One such set is $\{2, 3, 4\}$, where each element collectively dominates all vertices in the power graph of $\mathbb{Z}_6$. However, this set does not form the minimum dominating set. The existence of such a set highlights that minimum dominating sets are not necessarily unique. However, the minimum dominating set, which has the smallest possible cardinality, is distinct and optimal in terms of vertex coverage (Malarvizhi & Revathi, 2017).

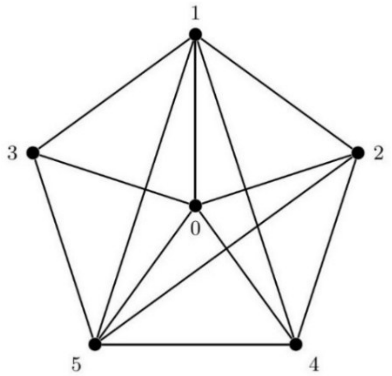

Figure 1. The Power graph of $\mathbb{Z}_6$ with $\{1\}$ or $\{3\}$ as minimal dominating set

For the case of $n = 8$, the cyclic group $\mathbb{Z}_8 = \{0, 1, 2, 3, 4, 5, 6, 7\}$ represents an example of a prime-power-order group. The corresponding power graph $\mathcal{P}(\mathbb{Z}_8)$ is depicted in Figure 4. Because $8 = 2^3$ is a power of a prime, Theorem 4 implies that every vertex belongs to the dominating set, and each vertex individually forms a minimal dominating set. Hence, removing any single vertex from $\mathcal{P}(\mathbb{Z}_8)$ still produces a connected graph.

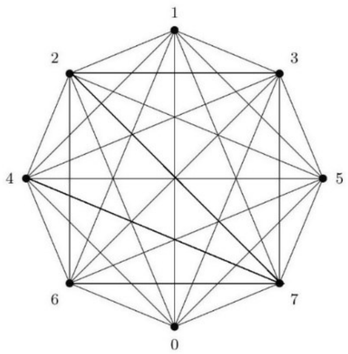

Figure 4. The Power graph of $\mathbb{Z}_8$

Discussion

The results presented in this study demonstrate how the structure of finite groups directly governs the domination and connectivity properties of their power graphs. For symmetric groups S_n, the results confirm that the identity element is a dominating vertex in the sense of Definition 3, as it is adjacent to every other element in the power graph. Lemma 1 shows that removing this vertex disconnects the graph, consistent with Definition 5, since paths between elements of

Leibniz: Jurnal Matematika
Januari 2026
Vol. 6 No. 01 Hal. 29-38
e-ISSN: 2775-2356
https://ejurnal.unisap.ac.id/leibniz/index

distinct cyclic subgroups rely on the identity as a common neighbor. This highlights the central structural role of the identity element in ensuring global connectivity in the power graph of S_n.

More significantly, Theorem 2 establishes the existence of a maximal dominating set, as defined in Definition 6, that excludes the identity element and whose removal does not disconnect the power graph. This result demonstrates that domination and connectivity are not solely dependent on the identity vertex. By selecting generators from each cyclic subgroup, the constructed dominating set satisfies Definition 4, as every vertex remains adjacent to at least one selected element. After removal, connectivity is preserved through the remaining identity vertex and internal adjacency within cyclic subgroups, showing that the power graph of S_n possesses a form of structural redundancy

In the case of cyclic groups $\mathbb{Z}_n$, Theorem 3 aligns directly with Definitions 2 and 4 by showing that generators dominate the power graph through their ability to generate the entire group. The result that any single generator forms a minimum dominating set reflects the strong internal cohesion of cyclic groups, where power relations naturally produce complete adjacency patterns. The existence of other minimal, but not minimum, dominating sets further illustrates that domination in power graphs need not be unique, even though optimal domination can be clearly characterized.

Taken together, these findings advance the theory of power graphs by demonstrating that connectivity can be resilient under the removal of dominating sets, even in highly structured non-abelian groups. This insight contributes a new dimension to the study of algebraic graphs, suggesting that domination-based robustness is an intrinsic feature tied to subgroup structure rather than merely to individual vertices. The results open avenues for further investigation into domination-resilient connectivity in other families of groups and related graph constructions, such as proper and enhanced power graphs.

CONCLUSION

This study investigated the structure of dominating sets in power graphs of symmetric groups S_n and cyclic groups $\mathbb{Z}_n$, focusing on the connectivity of these graphs after the removal of a dominating set. The findings confirm that power graphs of finite groups remain connected even after removing a dominating set, demonstrating their inherent structural resilience. Specifically, a maximal dominating set D_{max} was constructed for $\mathcal{P}(S_n)$, excluding the identity element, and its removal was shown not to disconnect the graph. Similarly, the role of dominating sets in $\mathcal{P}(\mathbb{Z}_n)$ was analyzed, highlighting their contribution to maintaining connectivity.

These results contribute to the broader understanding of domination and connectivity in power graphs, extending previous studies on proper power graphs and domination properties in algebraic structures. The findings have potential applications in network resilience, combinatorial optimization, and computational algebra, where understanding dominance and connectivity is essential. Future research could explore domination properties in other non-abelian groups, analyze the properties of the power graph after removing the dominating set, and examine the role of dominating sets in infinite groups or topological semigroups.

ACKNOWLEDGEMENTS

The author would like to acknowledge UIN Mahmud Yunus Batusangkar and Universiti Pendidikan Sultan Idris for the facilities provided.

REFERENCES

Aalipour, G., Akbari, S., Cameron, P. J., Nikandish, R., & Shaveisi, F. (2017). On the structure of the power graph and the enhanced power graph of a group. *Electronic Journal of Combinatorics*, *24*(3), 1–22. https://doi.org/10.37236/6497

Abawajy, J., Kelarev, A., & Chowdhury, M. (2013). Power graphs: A survey. *Electronic Journal of Graph Theory and Applications*, *1*(2), 125–147. https://doi.org/10.5614/ejgta.2013.1.2.6

Asmarani, E. Y., Syarifudin, A. G., Wardhana, I. G. A. W., & Switrayni, N. W. (2022). The Power Graph of a Dihedral Group. *Eigen Mathematics Journal*, *4*(2), 80–85. https://doi.org/10.29303/emj.v4i2.117

Cameron, P. J., & Ghosh, S. (2011). The power graph of a finite group. In *Discrete Mathematics* (Vol. 311, Issue 13, pp. 1220–1222). Elsevier BV. https://doi.org/10.1016/j.disc.2010.02.011

Chakrabarty, I., Ghosh, S., & Sen, M. K. (2009). Undirected power graphs of semigroups. *Semigroup Forum*, *78*(3), 410–426. https://doi.org/10.1007/s00233-008-9132-y

Chattopadhyay, S. (2019). Vertex connectivity of the power graph of a finite cyclic group. *Discrete Applied Mathematics*, *266*, 259–271. https://doi.org/10.1016/j.dam.2018.06.001

Chattopadhyay, S. (2020). Erratum: Vertex connectivity of the power graph of a finite cyclic group II. *Journal of Algebra and Its Applications*, *19*(2). https://doi.org/10.1142/S0219498820920012

Davvaz, B. (2021). A First Course in Group Theory. In *A First Course in Group Theory*. https://doi.org/10.1007/978-981-16-6365-9

Doostabadi, A., & Erfanian, A. (2013). *Some Results on Power Graph of Groups*. *August*, 547–550.

Doostabadi, A., & Ghouchan, M. F. D. (2015). On the Connectivity of Proper Power Graphs of Finite Groups. *Communications in Algebra*, *43*(10), 4305–4319. https://doi.org/10.1080/00927872.2014.945093

Hamzeh, A. (2021). On the Automorphisms Group of Finite Power Graphs. *Facta Universitatis, Series: Mathematics and Informatics*, *36*(1), 119. https://doi.org/10.22190/fumi200322010h

Haynes, T. W., Hedetniemi, S. T., & Henning, M. A. (2023). Domination in Graphs: Core Concepts. In *Springer Monographs in Mathematics*. https://doi.org/10.1007/978-3-031-09496-5

Haynes, T. W., Hedetniemi, S. T., & Slater, P. J. (1998). *Fundamentals of Domination in Graph*. Marcel Dekker, Inc.

Kelarev, A. V., & Quinn, S. J. (2002). Directed graphs and combinatorial properties of semigroups. *Journal of Algebra*, *251*(1), 16–26. https://doi.org/10.1006/jabr.2001.9128

Malarvizhi, D., & Revathi, V. (2017). A Review on Graphs with Unique Minimum Dominating Sets. *International Journal of Mathematics Trends and Technology (IJMTT)*, *44*(1), 32–37. https://doi.org/10.14445/22315373/ijmtt-v44p505

Manju, M. (2025). A Survey on Properties of Some Power Graphs. *International Journal For Multidisciplinary Research*, *7*(4), 1–6. https://doi.org/10.36948/ijfmr.2025.v07i04.52115

Marcus, D. A. (2020). *Graph Theory* (Vol 53). American Mathematical Society.

Mehranian, Z., Gholami, A., & Ashrafo, A. . . (2016). A Note On The Power Graph of a Finite Group. *International Journal of Group Theory*, *5*(1), 1–10. https://doi.org/10.1007/s11587-020-00520-w

Panda, R. P., & Krishna, K. V. (2018). On connectedness of power graphs of finite groups. *Journal of Algebra and Its Applications*, *17*(10), 1–20. https://doi.org/10.1142/S0219498818501840

Parveen, Manisha, & Kumar, J. (2024). *On the minimal (edge) connectivity of graphs and its applications to power graphs of finite groups*. 1–6. http://arxiv.org/abs/2408.10606

Pourghobadi, K., & Jafari, S. H. (2018). The diameter of power graphs of symmetric groups. *Journal of Algebra and Its Applications*, *17*(12), 1–11. https://doi.org/10.1142/S0219498818502341

S. Shukla, V. T. (2020). Domination and It'S Type in Graph Theory. *Journal of Emerging Technologies and Innovative Research*, *7*(3), 1549–1557. http://www.jetir.org/papers/JETIR2003225.pdf